Pulsed Magnet Crimping and Quarter Crushing

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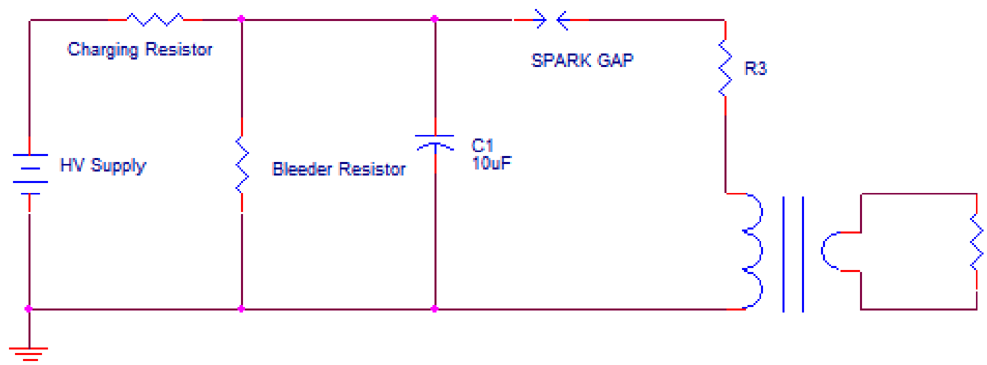

Figure 1: Schematic diagram of the pulse crimping setup. For simplicity, values for the charging resistor and bleeder resistor are omitted in this discussion. The work coil is shown as a transformer, and the single turn secondary is the work piece.

Magneforming, occasionally spelled magnetoforming, and even less commonly, EM forming is a metal fabrication technique that has been in use for many decades. It excels in applications where repeatability and reliability are necessary. The speed of the process is also occasionally helpful to maintain certain material properties in the work piece. In magneforming, electromagnetic forces are used to deform or otherwise bend a work piece. Recreationally, this process is used to crush or rip apart cans, to explode wires, and also quite interestingly, quarter crushing.

A schematic of such a setup is shown in Figure 1. A large capacitor bank is used to store energy. The capacitor's energy is dumped into a work coil with a high voltage switch, typically a spark gap, though ignitrons, thyratrons, and other switch technologies are used for this purpose. The capacitor's stored charge ends up as a large current the work coil. The magnetic field resulting from the current interacts with the work piece. In this discussion, we will first consider the work piece to be a thin aluminum tube, then later a quarter-sized slug of metal. The work coil's magnetic field induces a current in the work piece, which creates a magnetic field of its own in opposition to the work coil's field. Owing to transformer coupling between the work coil and the work piece, the current in the workpiece is typically much, much higher than the work coil's current. The magnetic fields then exert a force on each other. In adherence with Newton, both the work coil and the work piece feel this force. The force is radially directed – the work piece experiences a radial inward force, and the work coil experiences radial outward forces. In the work coil, this force ends up as hoop stress in the coil, and in many cases the coils are extremely heavily reinforced to withstand this hoop stress. Typically work coils are made of copper, which inherently has low ultimate yield strength and experiences work hardening very badly in this application, so composite coil assemblies are the norm. In the work piece, radial pressure is exerted on the tube, crimping a ferrule or plug into the tube. In the ATLAS detector in the LHC, the muon drift tubes were assembled at the University of Michigan, and were all crimped in this manner. This writeup, in fact, was inspired by a question that came up on an exam during my time in Physics grad school there.

Crimping Effect

Consider the aluminum tube to be a stack of infinitesimal loops. That allows us to calculate the current induced in the work piece. The capacitor sources a large current, which creates a large induction B in the work coil with a large $\frac{di}{dt}$. This creates a large $\frac{dB}{dt}$, which we know from Faraday induces an electric field,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

So an E field is created whose direction is along the circumference of the tube, which leads to an emf in the tube. For simplicity, we can assume the coil and the tube are nearly the same size so all of the field from the coil goes through the tube, and no stray flux is wasted. The emf would then have the following form:

$$\mathcal{E} = -\mathcal{M}\frac{di}{dt}$$

Where i is the changing current in the inductor, and $\mathcal{M}$ the mutual inductance of the system of coil and work piece. This mutual inductance is difficult to calculate, but the effect of the current is clear. The emf and the resistance of the work piece gives rise to an induced current in the work piece. This current is in the opposite direction of the current in the work coil. This current creates a B field in the opposite direction of work coil's B field – Lenz's Law. The two opposing inductions exert repulsive forces on each other. The solenoid and the tube both feel this repulsive force. The direction of the force is radial inward for the tube, and radially outward for the work coil.

Circuit Analysis

The schematic as drawn has two Kirchoff's loops. But the transient response we are looking for is what happenes after the capacitor is fully charged. Let's assume the charging resistor is 100kΩ, and the bleeder resistor is 10MΩ. So we'll look at this in two steps. First, we know that the steady state voltage of the capacitor will be the voltage divider potential of the charging resistor and the bleeder resistor. So in this case, the capacitor voltage will be

$$V_{capacitor} = \frac{10M\Omega \cdot 10kV}{(100k\Omega + 10M\Omega)} = 9.9kV$$

This means that we have a capacitor charged to steady state at 9.9kV. For completeness' sake we can say that the high voltage power supply can be replaced with a Thevenin equivalent source with 10kV voltage, and a resistance of 100kΩ - which during the closing of the power switch will limit the current of the HV supply to 10kV/100k, or 0.1A. This 0.1A will be dwarfed by the main switch current, and so will be ignored. I will use the voltage rule, as it's a little easier to see. One could argue that there are three loops actually, including the transformer action and the work piece, but that's going a bit over the line, even for me.

$$V_0 - R_s i - \frac{1}{C}\int_0^t i dt - L\frac{di}{dt} = 0$$

Assuming that the boundary condition i = 0 at t = ∞ we have

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

which has a familiar general form:

$$i(t) = e^{\frac{-R_s t}{2L}}\frac{2V_0}{\omega_0 L}\sinh\left(\frac{R_s t}{2L}\sqrt{1 - \frac{4L}{R_s^2 C}}\right)$$

Of course, this is unwieldy, but we have a trick up our sleeve. In real applications, the radical is imaginary, as $\frac{4L}{R^2 C} > 1$, which yields an even simpler form

$$i(t) = e^{\frac{-R_s t}{2L}}\frac{V_0}{\omega_0 L}\sin\omega_0 t$$

$$\omega_0 = \sqrt{\frac{1}{LC} - \frac{R_s^2}{4L^2}}$$

which is just an exponentially damped sinewave, as expected. Note the resonant frequency of the RLC system is as normal. An important thing to look at here is the damping. Generally speaking, in these systems the oscillations are either critically damped or somewhat overdamped. Otherwise they would oscillate forever. The damping constant is defined as

$$\zeta = \frac{R_s}{2}\sqrt{\frac{C}{L}}$$

and the system is critically damped if ζ = 1, and underdamped so long as ζ >1.

To characterize the circuit and put numbers to things, we must first look at the capacitor and the work coil. The capacitor C1 is 10uF and is charged to 10kV. This of course yields 500J of energy stored in the capacitor.

$$E = \frac{1}{2}CV^2$$

The impedance of the work coil (inductance and resistance both) is complicated by the work piece. Transformer coupling between the work coil and the work piece yields an effective inductance that is the combination of the work piece inductance and the work coil inductance seen by the capacitor. Instead of calculating the inductance from first principles, typically the inductance would be measured with the work piece in place. Geometric factors drastically affect the value of the inductance, which in turn affects the peak current, which in turn affects the peak pressures reached. For this reason, in most industrial applications, a custom work coil is created for each individual process. Inductances are typically as low as practicable to reach the highest peak current and thus the highest peak pressures. In the exam problem posed, the inductance listed was 3µH. In practice this would be a rather large inductance compared to the work piece and coil system – that is by design. Generally speaking, there is a tuning inductance and a work coil and the tuning inductance does not participate in the action on the work piece. It is usually assumed that the work coil and workpiece present a very small inductance to the system, and the tuning inductor is what sets the inductive reactance of the system. Thus the inductive reactance of the work coil and the workpiece are small compared to the tuning inductance. This is done to limit the di/dt out of the capacitor and ultimately limit the overall current. If the inductance was left entirely to the work coil and work piece, the time-evolution of the sizes of the two would drastically complicate things, as well as put little limit on the current. Transformer coupling also complicates the effective R_s of the system – the workpiece resistance is reflected back to the work coil, altering the actual resistance in the circuit model. The series resistance quoted in the problem is 0.1Ω. The resonant frequency of the system described is then 29kHz.

To determine the peak current, we can go the hard way or the easy way. The hard way starts by taking the derivative of the current calculated above, and set it equal to zero, and solve for t. The derivative is just chain rule, then set to zero.

$$\frac{di(t)}{dt} = -\frac{R_s}{2L} e^{-\frac{R_s t}{2L}} \frac{V_0}{\omega_0 L} \sin \omega_0 t + e^{-\frac{R_s t}{2L}} \frac{V_0}{L} \sin \omega_0 t = 0;$$

$$\frac{R_s}{2V_0\omega_0} \cdot \frac{\sin \omega_0 t}{\cos \omega_0 t} = 1; \quad \tan \omega_0 t = \frac{2V_0\omega_0}{R_s}$$

You get the following recursion formula for the zeros (maximum current peaks):

$$t_{max} = \frac{1}{\omega} \tan^{-1} \frac{2V_0\omega_0}{R_s}$$

Plugging in the numbers, we get t_{max} = 8.63µs. Lots of algebra.

The fast way is to remember that the current is going to be a sinewave in an RLC circuit. We already calculated the resonant frequency. So, the first peak in the current is at ¼ of the period of oscillation, or 4/29000 = 8.63us. Plugging that time into the current relation we got gives us a current peak of 16kA.

An important thing to note here is the skin depth of the material of the work piece. The skin depth arises from the damping of EM waves in dispersive media – and is just exponential decay in amplitude along the direction of travel. The skin depth is defined as the distance where 1/e of the amplitude is lost. This distance is just

$$\delta = \frac{c}{\sqrt{2\pi\mu\mu_0\omega_0\sigma}}$$

For our setup, the material is just aluminum which has a relative permeability of 1, and a resistivity of 2.8×10^{-8} Ωm, the skin depth is then 0.5mm. That means 64% of the force is in the outer 0.5mm.

Pressure

The magnetic pressure, or the force exerted by the work coil can be derived many ways. I am outlining two derivations, since the ultimate answer is a little surprising. We'll start with the virtual work method.

$$U_{magnetic} = \int \frac{B^2}{2\mu_0} d\tau = \frac{B^2}{2\mu_0}(volume) = \frac{B^2}{2\mu_0} 2\pi r l$$

assuming a cylindrical arrangement. Force is just the minus gradient of energy, so we define a magnetic force thusly:

$$F_{magneitc} = -\nabla U = -\frac{dU_{magnetic}}{dr}\hat{r} = -\frac{B^2}{2\mu_0} 2\pi r l$$

Radially inward. The force defined in this manner pushes inward on the tube, crushing it. Pressure, then, is the force over the surface it's acting on:

$$\not{p}_{magnetic} = \frac{|F_{magnetic}|}{area} = \frac{|F_{magnetic}|}{2\pi r l} = \frac{B^2}{2\mu_0}$$

Huh? What just happened there? It's independent of the dimensions. Now that's a bit of a cheat, since the dimensions end up determining B to a great extent, but the pressure isn't dependent on the size or shape or anything. Just the square of the induction B. Can't be right.

Now we try with the stress tensor

$$T_{ij} = \epsilon_0 \left(E_i E_j - \frac{1}{2} E^2 \delta_{ij} \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} B^2 \delta_{ij} \right)$$

Now in our system, i = j, and we are only looking for magnetic pressure, so Tij reduces to

$$T_{i=j} = -\frac{B}{2\mu_0}$$

Ut oh. This looks fishy. Force is the time derivative of momentum of a field, so

$$\frac{dp_{i=j}}{dt} = \sum_j \int_v \frac{d}{dx_j} T_{i=j} d^3x$$

Which with a divergence theorem trick ends up as

$$F_{magnetic} = \int_s T_{i=j} \hat{n}\, da = -\frac{B^2}{2\mu_0}$$

Ok, well I guess that was right all along. Weird. So with a solenoid work coil, B is simply calculated

$$\vec{B} = \mu_0 n i(t)$$

Where n is N/l. Here the field is only dependent on the number of turns and the length of the coil. Now the pressure as a function of time is pretty simple

$$\not{p}_{magnetic} = \frac{\mu_0}{2} n^2 i^2 (t_{max})$$

$$\not{p}_{magnetic} = \frac{\mu_0}{2} \left[\frac{NV_0}{l\omega_0 L}\right]^2 \left(e^{\frac{-R_s t_{max}}{2L}} \sin \omega_0 t_{max}\right)^2$$

but we already know that the fast way to get to the t_{max} is to take the period of the oscillation (easy to calculate) and divide it by 4.

$$t_{max} = \frac{1}{4f} = \frac{\pi}{2}\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)^{-\frac{1}{2}}$$

of course, this makes the sin() term above disappear, as sin(pi/2) = 1. We are left with a somewhat more pleasing relationship

$$\not{p}_{magnetic} = \frac{\mu_0}{2} \left[\frac{NV_0}{l\omega_0 L}\right]^2 e^{-\frac{\pi R_s}{4L\omega_0}}$$

If we take our coil as 10 turns, 2cm diameter and 2cm long, we get max pressure of 43MPa = 6240psi. That's a lot of pressure. Since we know that an external tuning inductor is used and is in series with the work coil, the current in the work coil is still correct. The work coil inductance alone without the workpiece should be 1.3uH. The same coil with an Aluminum tube inserted with close tolerance would end up as around 0.33uH (from experimentation). It's interesting to note that the magnetic pressure developed depends on the size of the coil, the number of turns, and the current in it. The inductance of the total wiring and stray flux ends up changing the time to achieve peak current, but the inductance of the work coil is not explicitly the total inductance. This is why the tuning inductor is important in repeatability of the results.

Quarter Shrinking

Many more kA are typical in quarter shrinking than is typical with pulsed magnetic crimping operations. Here the work coil is intended to explode owing to hoop stress and partial melting of the wire. This is done to prevent ringing in the capacitor/inductor circuit, since the ringing through to reverse the voltage on the capacitor severely stresses the dielectric. Most high energy pulse caps have a rating of % reversal allowed during a shot.

It is quite valid to assume that in the typical arrangement used in quarter shrinking that there is close to an n:1 step up of current in the workpiece. I^2R heating of the material would suggest on the order of 1MA is flowing in the outer 1/e periphery of the quarter. The skin depth of the quarter is going to also be on the order 0.5mm, and the heat capacity of the quarter is dominated by the copper in today's copper-nickel clad coins. I will just claim that for these purposes the entire thing is copper. Then, we've got a "washer" of copper, 0.95" diameter, and let's say 2 e-foldings wide (0.25"), and 0.065" thick. This would have an approximate resistance of

$$R = \frac{l\rho}{A} = \frac{12cm \cdot 1.7\mu\Omega cm}{0.17cm \ \cdot 0.64cm} \sim 0.18m\Omega$$

If transformer action allows up to say 1MA in the coil, then we have 150-180MW available to heat the coin in the time the current is there. Most studies have the coin shrinking done and the coil open in 20-

40us. In that case, the total applied joules as heat to the coin are then around 6kJ. If the quarter starts out at 30C, the material of the coin experiences some heat rise.

$$\Delta H = mC\Delta T$$
$$6000 = 5.67 \cdot .386\,\Delta T$$
$$\Delta T = 2741°K = 2448°C$$

So... what is the melting point of copper? 1085C. This is certainly hot enough to melt that part of the coin and allow it to experience Lorentz forces and Lenz law repulsion and crush the coin radially. Clearly not all the pulse energy ends up in the coin, as a fair amount ends up blowing up the work coil. Even if only half the energy ends as joule heat, it's still enough to locally melt the coin. These effects in combination certainly account for the fun shapes. It is not clear that the magnetic pressure alone without annealing and localized melting would be able to crush the quarters. I would venture a guess that the magnetic pressure in a given quarter shrinking setup could easily be an order of magnitude higher than in the problem solved above. However, ultimate yield in copper is around 30kPsi, and yield strength is only 5kPsi. So it is possible that copper could bend nicely even with the modest 500J pulse in the example above, and little induced joule heating. Looking at the coins themselves, the center of the coin, which doesn't feel much joule heating, but feels all of the pressure, appear to be just radially squashed. The outer rim of the coin could potentially feel quite a lot of joule heating and appreciable melting. It stands to reason that the oddly squished, maybe even toroidal shapes make some sense.